# DÉRIVÉES ET APPLICATIONS

Dérivée de fonctions

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https://bit.ly/4jGC0ij

QVC

Calcule la dérivée de chaque fonction et donne-la, si possible, sous forme factorisée et/ou simplifiée :

(1) $f(x) = x + \sqrt{x} + \sqrt[3]{x} = x + \sqrt{x} + x^{1/3}$

$f'(x) = 1 + \frac{1}{2\sqrt{x}} + \frac{1}{3} \cdot x^{-2/3}$

$= 1 + \frac{1}{2\sqrt{x}} + \frac{1}{3 \cdot \sqrt[3]{x^2}}$

(2) $f(x) = \frac{\sqrt[5]{x}}{5} = \frac{1}{5} \cdot x^{1/5}$

$f'(x) = \frac{1}{5} \cdot \frac{1}{5} \cdot x^{-4/5}$

$= \frac{1}{25\sqrt[5]{x^4}}$

(3) $f(x) = (3x-5)^3$

$f'(x) = 3 \cdot (3x-5)^2 \cdot 3$

$= 9 \cdot (3x-5)^2$

(4) $f(x) = \frac{2x-1}{x+3}$

$f'(x) = \frac{2 \cdot (x+3) - (2x-1) \cdot 1}{(x+3)^2} = \frac{2x+6-2x+1}{(x+3)^2} = \frac{7}{(x+3)^2}$

(5) $f(x) = \frac{1-\sin x}{1+\cos x}$

$f'(x) = \frac{-\cos x \cdot (1+\cos x) - (1-\sin x) \cdot (-\sin x)}{(1+\cos x)^2}$

$= \frac{-\cos x - \cos^2 x + \sin x - \sin^2 x}{(1+\cos x)^2}$

$= \frac{-1 - \cos x + \sin x}{(1+\cos x)^2}$

(6) $f(x) = \dfrac{2\sin x}{\sin x - \cos x}$

$$f'(x) = \frac{2\cos x.(\sin x - \cos x) - 2\sin x.(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{\cancel{2\cos x.\sin x} - 2\cos^2 x - \cancel{2\sin x.\cos x} - 2\sin^2 x}{(\sin x - \cos x)^2}$$

$$= \frac{-2}{(\sin x - \cos x)^2}$$

(7) $f(x) = \dfrac{x^2 + 2x}{1 - x^2}$

$$f'(x) = \frac{(2x+2).(1-x^2) - (x^2+2x).(-2x)}{(1-x^2)^2}$$

$$= \frac{2x - \cancel{2x^3} + 2 - 2x^2 + \cancel{2x^3} + 4x^2}{(1-x^2)^2}$$

$$= \frac{2x^2 + 2x + 2}{(1-x^2)^2}$$

(8) $f(x) = \sin^2 x - 2.\cos^3 x$

$$f'(x) = 2.\sin x.\cos x - 2.3.\cos^2 x.(-\sin x)$$

$$= 2\sin x.\cos x + 6\sin x.\cos^2 x$$

$$= 2\sin x.\cos x.(1 + 3\cos x)$$

(9) $f(x) = \sqrt{1 + 2x - x^2}$

$$f'(x) = \frac{2 - 2x}{2\sqrt{1+2x-x^2}} = \frac{1-x}{\sqrt{1+2x-x^2}}$$

(10) $f(x) = x^2.\sqrt{1+x^2}$

$$f'(x) = 2x.\sqrt{1+x^2} + x^2.\frac{2x}{2\sqrt{1+x^2}}$$

$$= 2x.\sqrt{1+x^2} + \frac{x^3}{\sqrt{1+x^2}}$$

$$= \frac{2x.(1+x^2) + x^3}{\sqrt{1+x^2}}$$

$$= \frac{2x + 2x^3 + x^3}{\sqrt{1+x^2}}$$

$$= \frac{3x^3 + 2x}{\sqrt{1+x^2}}$$

(11) $f(x)=\dfrac{2x^4+3x^2-1}{x^2}$

$$f'(x)=\frac{(8x^3+6x).x^2-(2x^4+3x^2-1).2x}{x^4}$$

$$=\frac{8x^5+6x^3-4x^5-6x^3+2x}{x^4}$$

$$=\frac{4x^5+2x}{x^4}$$

$$=\frac{4x^4+2}{x^3}$$

(12) $f(x)=\dfrac{6}{(3x^2-\pi)^4}$ $=6.(3x^2-\pi)^{-4}$

$$f'(x)=6.(-4).(3x^2-\pi)^{-5}.6x$$

$$=\frac{-144x}{(3x^2-\pi)^5}$$

(13) $f(x)=2x-\dfrac{4}{\sqrt{x}}$ $=2x-4.x^{-1/2}$

$$f'(x)=2-4.-\frac{1}{2}x^{-3/2}$$

$$=2+\frac{2}{\sqrt{x^3}}$$

(14) $f(x)=\dfrac{1}{\sqrt[3]{x}}$ $=x^{-1/3}$

$$f'(x)=-\frac{1}{3}.x^{-4/3}=\frac{-1}{3.\sqrt[3]{x^4}}$$

(15) $f(x)=(x^6+1)^5.(4x+7)^3$

$$f'(x)=5.(x^6+1)^4.6x^5.(4x+7)^3+(x^6+1)^5.3.(4x+7)^2.4$$

$$=6.(x^6+1)^4.(4x+7)^2.\left[5x^5(4x+7)+2.(x^6+1)\right]$$

$$=6.(x^6+1)^4.(4x+7)^2.(20x^6+35x^5+2x^6+2)$$

$$=6.(x^6+1)^4.(4x+7)^2.(22x^6+35x^5+2)$$

(16) $f(x)=\dfrac{5x^2-7x}{x^2+2}$

$$f'(x)=\frac{(10x-7)(x^2+2)-(5x^2-7x).2x}{(x^2+2)^2}$$

$$=\frac{\cancel{10x^3}+20x-7x^2-14\cancel{-10x^3}+14x^2}{(x^2+2)^2}$$

$$=\frac{7x^2+20x-14}{(x^2+2)^2}$$

(17) $f(x)=3.\sin^3(3x+3)$

$$f'(x)=3.3.\sin^2(3x+3).\cos(3x+3).3$$

$$=27.\sin^2(3x+3).\cos(3x+3)$$

(18) $f(x)=(3x^2-2).(4x^3+1)$

$$f'(x)=(6x).(4x^3+1)+(3x^2-2).(12x^2)$$

$$=6x.\left[4x^3+1+2x.(3x^2-2)\right]$$

$$=6x.(4x^3+1+6x^3-4x)$$

$$=6x.(10x^3-4x+1)$$

(19) $f(x)=\dfrac{x+6}{x^3}$

$$f'(x)=\frac{1.x^3-(x+6).3x^2}{x^6}$$

$$=\frac{x^3-3x^3-18x^2}{x^6}$$

$$=\frac{-2x^3-18x^2}{x^6}=\frac{-2x^2.(x+9)}{x^6}=\frac{-2.(x+9)}{x^4}$$

(20) $f(x)=(2x+1).(3x-2).(3x+3)$

$$=(6x^2-4x+3x-2).(3x+3)=(6x^2-x-2).(3x+3)$$

$$=18x^3+18x^2-3x^2-3x-6x-6=18x^3+15x^2-9x-6$$

$$f'(x)=54x^2+30x-9$$