DÉRIVÉES ET APPLICATIONS

Dérivée de fonctions

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https://bit.ly/4jGC0ij

QVC

Calcule la dérivée de chaque fonction et donne-la, si possible, sous forme factorisée et/ou simplifiée :

(1) $f(x)=4x^3-6x^2+x+2$

$f'(x)=12x^2-12x+1$

(2) $f(x)=\dfrac{1}{\sqrt{x^5}} = x^{-\frac{5}{2}}$

$f'(x)=-\frac{5}{2}\cdot x^{-\frac{7}{2}} = \dfrac{-5}{2\sqrt{x^7}}$

(3) $f(x)=\dfrac{3x-5}{2x+3}$

$f'(x)=\dfrac{3\cdot(2x+3)-(3x-5)\cdot 2}{(2x+3)^2} = \dfrac{6x+9-6x+10}{(2x+3)^2} = \dfrac{19}{(2x+3)^2}$

(4) $f(x)=(x^2+3)(2x^3-1)$

$f'(x)=2x\cdot(2x^3-1)+(x^2+1)\cdot 6x^2$

$= 2x\cdot(2x^3-1+(x^2+1)\,3x)$

$= 2x\cdot(2x^3-1+3x^3+3x)$

$= 2x\cdot(5x^3+3x-1)$

(5) $f(x)=3x^4+\sqrt[3]{x^2}-\dfrac{1}{x} = 3\cdot x^4+x^{\frac{2}{3}}-\frac{1}{x}$

$f'(x)=12x^3+\frac{2}{3}\cdot x^{-\frac{1}{3}}-\left(\frac{-1}{x^2}\right)$

$=12x^3+\dfrac{2}{3\cdot\sqrt[3]{x}}+\dfrac{1}{x^2}$

(6) $f(x)=\dfrac{3}{x^2-x} = 3 \cdot \dfrac{1}{x^2-x}$

$f'(x) = 3 \cdot \dfrac{-(x^2-x)'}{(x^2-x)^2} = 3 \cdot \dfrac{-(2x-1)}{(x^2-x)^2} = \dfrac{-6x-3}{(x^2-x)^2}$

(7) $f(x)=\dfrac{x^2-3x}{4x+5}$

$f'(x) = \dfrac{(2x-3)(4x+5)-(x^2-3x) \cdot 4}{(4x+5)^2} = \dfrac{8x^2+10x-12x-15-4x^2+12x}{(4x+5)^2}$

$= \dfrac{4x^2+10x-15}{(4x+5)^2}$

(8) $f(x)=(x^3+1)^5$

$f'(x) = 5 \cdot (x^3+1)^4 \cdot (3x^2) = 15x^2 \cdot (x^3+1)^4$

(9) $f(x)=\sqrt{x^4+2}$

$f'(x) = \dfrac{4x^3}{2\sqrt{x^4+2}} = \dfrac{2x^3}{\sqrt{x^4+2}}$

(10) $f(x)=\sqrt[3]{(x^2-4x+3)^2} = (x^2-4x+3)^{2/3}$

$f'(x) = \dfrac{2}{3} \cdot (x^2-4x+3)^{-1/3} \cdot (2x-4)$

$= \dfrac{2 \cdot (2x-4)}{3 \sqrt[3]{x^2-4x+3}}$

(11) $f(x)=\dfrac{(3x+2)^2}{x-1}$

$f'(x) = \dfrac{2 \cdot (3x+2) \cdot 3 \cdot (x-1) - (3x+2)^2 \cdot 1}{(x-1)^2}$

$= \dfrac{(3x+2) \cdot (6 \cdot (x-1) - (3x+2))}{(x-1)^2}$

$= \dfrac{(3x+2) \cdot (6x-6-3x-2)}{(x-1)^2} = \dfrac{(3x+2) \cdot (3x-8)}{(x-1)^2}$

(12) $f(x)=\frac{(x+2)^3}{(2x+4)^2}$

$$f'(x)=\frac{3.(x+2)^2.(2x+4)^2-(x+2)^3.2.(2x+4).2}{(2x+4)^4}$$

$$=\frac{(x+2)^2.\cancel{(2x+4)}\,[3.(2x+4)-4.(x+2)]}{(2x+4)^{\cancel{4}\,3}}$$

$$=\frac{(x+2)^2.(2x+4)}{(2x+4)^3}$$

$$=\frac{(x+2)^2}{2x+4}$$

(13) $f(x)=\frac{x}{(4-x^2)^3}$

$$f'(x)=\frac{1.(4-x^2)^3-x.3.(4-x^2)^2.(-2x)}{(4-x^2)^6}$$

$$=\frac{\cancel{(4-x^2)^2}.[4-x^2+6x^2]}{(4-x^2)^{\cancel{6}\,4}}$$

$$=\frac{5x^2+4}{(4-x^2)^4}$$

(14) $f(x)=\sqrt{x}.(5x+6)^3$

$$f'(x)=\frac{1}{2\sqrt{x}}.(5x+6)^3+\sqrt{x}.\,3.(5x+6)^2.5$$

$$=(5x+6)^2.\left(\frac{5x+6}{2\sqrt{x}}+15\sqrt{x}\right)=(5x+6)^2.\left(\frac{5x+6}{2\sqrt{x}}+\frac{15\sqrt{x}.2\sqrt{x}}{2\sqrt{x}}\right)$$

$$=(5x+6)^2.\left(\frac{5x+6+30x}{2\sqrt{x}}\right)=(5x+6)^2.\left(\frac{35x+6}{2\sqrt{x}}\right)$$

(15) $f(x)=\sin(x^2)$

$$f'(x)=\cos(x^2).\,2x$$

(16) $f(x)=\cos(\sqrt{x})$

$$f'(x)=-\sin(\sqrt{x}).\frac{1}{2\sqrt{x}}$$

(17) $f(x)=\dfrac{\cos x-1}{\sin x-1}$

$$f'(x)=\frac{(-\sin x).(\sin x-1)-(\cos x-1).\cos x}{(\sin x-1)^2}$$

$$=\frac{-\sin^2 x+\sin x-\cos^2 x+\cos x}{(\sin x-1)^2}$$ $-(\sin^2 x+\cos^2 x)=-1$

$$=\frac{-1+\sin x+\cos x}{(\sin x-1)^2}$$

(18) $f(x)=\cos x.\sin x$

$$f'(x)=-\sin x\,.\,\sin x+\cos x.\cos x$$

$$=-\sin^2 x+\cos^2 x$$

(19) $f(x)=\dfrac{\sin x}{\cos x+1}$

$$f'(x)=\frac{\cos x.(\cos x+1)-\sin x.(-\sin x)}{(\cos x+1)^2}$$

$$=\frac{\cos^2 x+\cos x+\sin^2 x}{(\cos x+1)^2}$$

$$=\frac{1+\cos x}{(\cos x+1)^2}=\frac{1}{\cos x+1}$$

(20) $f(x)=2.\sin\left(3x-\dfrac{\pi}{2}\right)+1$

$$f'(x)=2.\cos\left(3x-\frac{\pi}{2}\right).\underbrace{\left(3x-\frac{\pi}{2}\right)'}_{=3}$$

$$=6.\cos\left(3x-\frac{\pi}{2}\right)$$