

UAA 3 : Asymptotes et limites

Solutions

A. Limites en un réel

5. Exercices

1. Calcule les limites suivantes (en distinguant éventuellement limite à gauche et limite à droite) et donnes-en une interprétation graphique :

$$(1) \lim_{x \rightarrow 3} (x^2 - 4x) = -3 \rightarrow (3; -3) \in G_f$$

$$(2) \lim_{x \rightarrow 2} (4x^2 - 8x) = 0 \rightarrow (2; 0) \in G_f$$

$$(3) \lim_{x \rightarrow 3} \frac{x^2 + 6x + 5}{x - 3} = \begin{cases} \nearrow \lim_{x \rightarrow 3^-} f(x) = -\infty \\ \searrow \lim_{x \rightarrow 3^+} f(x) = +\infty \end{cases} \rightarrow AV \equiv x = 3$$

$$(4) \lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x^2 - 2x - 3} = -1 \rightarrow \text{Trou en } (-1; -1)$$

$$(5) \lim_{x \rightarrow 1} \frac{-x^2 + 2x - 1}{x^3 - 1} = 0 \rightarrow \text{Trou en } (1; 0)$$

$$(6) \lim_{x \rightarrow -2} \frac{x^5 + x^4 - 2x^3 + 2x^2 + 10x + 12}{2x^3 + x^2 - 11x - 10} = \frac{26}{9} \rightarrow \text{Trou en } \left(-2; \frac{26}{9}\right)$$

$$(7) \lim_{x \rightarrow -2} \frac{1 + \frac{4}{3x+2}}{x^3 + 8} = -\frac{1}{16} \rightarrow \text{Trou en } \left(-2; -\frac{1}{16}\right)$$

$$(8) \lim_{x \rightarrow 2} \frac{x^3 - 2x^2 + 3x - 6}{x^4 - 5x^2 + 4} = \frac{7}{12} \rightarrow \text{Trou en } \left(2; \frac{7}{12}\right)$$

$$(9) \lim_{x \rightarrow 2} \frac{x-6}{x^2-4} = \begin{cases} \nearrow \lim_{x \rightarrow 2^-} f(x) = +\infty \\ \searrow \lim_{x \rightarrow 2^+} f(x) = -\infty \end{cases} \rightarrow AV \equiv x = 2$$

$$(10) \lim_{x \rightarrow 2} \frac{2x-1}{\sqrt{4x+1}-\sqrt{6x}} = -3-2\sqrt{3} \rightarrow \text{Trou en } (2; -3-2\sqrt{3})$$

$$(11) \lim_{x \rightarrow -1} \frac{x+1}{\sqrt{x+5}-2} = 4 \rightarrow \text{Trou en } (-1; 4)$$

$$(12) \lim_{x \rightarrow 4} \frac{\sqrt{x+5}-3}{x-4} = \frac{1}{6} \rightarrow \text{Trou en } \left(4; \frac{1}{6}\right)$$

$$(13) \lim_{x \rightarrow -1} \frac{\sqrt{2-x}-\sqrt{x+4}}{\sqrt{-3x}-\sqrt{2x+5}} = \frac{2}{5} \rightarrow \text{Trou en } \left(-1; \frac{2}{5}\right)$$

$$(14) \lim_{x \rightarrow 3} \frac{|x-3| \cdot (x^2 - 4x + 5)}{2x^3 - 7x^2 + 7x - 12} = \begin{cases} \nearrow \lim_{x \rightarrow 3^-} f(x) = -\frac{2}{19} \\ \searrow \lim_{x \rightarrow 3^+} f(x) = \frac{2}{19} \end{cases} \rightarrow \text{Trous en } \left(3; \frac{2}{19}\right) \text{ et en } \left(3; -\frac{2}{19}\right)$$

$$(15) \lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 - 4} = \frac{5}{4} \rightarrow \text{Trou en } \left(2; \frac{5}{4}\right)$$

$$(16) \lim_{x \rightarrow 1} \frac{\sqrt{2x^2+7x-3}}{\sqrt{x^2-2x+1}} = \begin{cases} \nearrow \lim_{x \rightarrow 1^-} f(x) = -\frac{11}{6} \\ \searrow \lim_{x \rightarrow 1^+} f(x) = \frac{11}{6} \end{cases} \rightarrow \text{Trous en } \left(1; \frac{11}{6}\right) \text{ et en } \left(1; -\frac{11}{6}\right)$$

$$(17) \lim_{x \rightarrow 3} \frac{\sqrt{2x+3}-x}{2-\sqrt{x+1}} = \frac{8}{3} \rightarrow \text{Trou en } \left(3; \frac{8}{3}\right)$$

$$(18) \lim_{x \rightarrow 1} \frac{x^3 + 3x^2 - x - 3}{x^3 - 3x + 2} = \begin{array}{l} \nearrow \lim_{x \rightarrow 1^-} f(x) = -\infty \\ \searrow \lim_{x \rightarrow 1^+} f(x) = +\infty \end{array} \Rightarrow AV \equiv x = 1$$

$$(19) \lim_{x \rightarrow 2} \frac{x^3 - 2x^2 - 4x + 8}{x^3 - 5x^2 + 8x - 4} = 4 \Rightarrow \text{Trou en } (2; 4)$$

$$(20) \lim_{x \rightarrow 3} \frac{(x^2 - 6x + 9)(x^2 - 5x + 4)}{(2x - 6)(5x - 5)} = 0 \Rightarrow \text{Trou en } (3; 0)$$

$$(21) \lim_{x \rightarrow -1} \frac{x^2 + 2x - 3}{x^3 + 2x^2 - 11x - 12} = \begin{array}{l} \nearrow \lim_{x \rightarrow -1^-} f(x) = -\infty \\ \searrow \lim_{x \rightarrow -1^+} f(x) = +\infty \end{array} \Rightarrow AV \equiv x = -1$$

$$(22) \lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x^2 + x - 6}} = 0 \Rightarrow \text{Trou en } (2; 0)$$