

UAA 5 : Fonctions trigonométriques

Solutions

G. Equations et inéquations trigonométriques

1. Equations

(1) Equations élémentaires

Exercices : Résous les équations suivantes. Exprime les angles en radians et détermine les solutions principales.



Tous les exercices sont à faire sans calculatrice !

Série 1 :

(1) $\sin x = 0$

$$x = k\pi$$

$$SP = \{0; \pi\}$$

(2) $\sin x = 1$

$$x = \frac{\pi}{2} + 2k\pi$$

$$SP = \left\{ \frac{\pi}{2} \right\}$$

(3) $\sin x = -1$

$$x = -\frac{\pi}{2} + 2k\pi$$

$$SP = \left\{ \frac{3\pi}{2} \right\}$$

(4) $\cos x = 0$

$$x = \frac{\pi}{2} + k\pi$$

$$SP = \left\{ \frac{\pi}{2}; \frac{3\pi}{2} \right\}$$

$$(5) \cos x = 1$$

$$x = 2k\pi$$

$$SP = \{0\}$$

$$(6) \cos x = -1$$

$$x = \pi + 2k\pi$$

$$SP = \{\pi\}$$

Série 2 :

$$(1) \sin(3x) = \sin\left(2x - \frac{\pi}{4}\right)$$

$$x = -\frac{\pi}{4} + 2k\pi \text{ ou } x = \frac{\pi}{4} + 2k\frac{\pi}{5}$$

$$SP = \left\{ \frac{7\pi}{4}; \frac{\pi}{4}; \frac{13\pi}{20}; \frac{21\pi}{20}; \frac{29\pi}{20}; \frac{37\pi}{20} \right\}$$

$$(2) \tan(\pi + x) = -\tan\left(\frac{\pi}{6} - x\right)$$

$$S = \emptyset$$

$$(3) \sin x = -\frac{\sqrt{3}}{2}$$

$$x = -\frac{\pi}{3} + 2k\pi \text{ ou } x = \frac{4\pi}{3} + 2k\pi$$

$$SP = \left\{ \frac{5\pi}{3}; \frac{4\pi}{3} \right\}$$

$$(4) \sin(4x) = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{16} + k\frac{\pi}{2} \text{ ou } x = \frac{3\pi}{16} + k\frac{\pi}{2}$$

$$SP = \left\{ \frac{\pi}{16}; \frac{9\pi}{16}; \frac{17\pi}{16}; \frac{25\pi}{16}; \frac{3\pi}{16}; \frac{11\pi}{16}; \frac{19\pi}{16}; \frac{27\pi}{16} \right\}$$

$$(5) \quad \cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3} + 2k\pi \text{ ou } x = -\frac{2\pi}{3} + 2k\pi$$

$$SP = \left\{ \frac{2\pi}{3}; \frac{4\pi}{3} \right\}$$

$$(6) \quad 1 + \sqrt{3} \tan x = 0$$

$$x = -\frac{\pi}{6} + k\pi$$

$$SP = \left\{ \frac{5\pi}{6}; \frac{11\pi}{6} \right\}$$

$$(7) \quad 2 \cdot \cos\left(x + \frac{\pi}{3}\right) - 1 = 0$$

$$x = 2k\pi \text{ ou } x = -\frac{2\pi}{3} + 2k\pi$$

$$SP = \left\{ 0; \frac{4\pi}{3} \right\}$$

$$(8) \quad 5 \cdot \cos\left(4x - \frac{\pi}{4}\right) + 5 = 0$$

$$x = \frac{5\pi}{16} + k \frac{\pi}{2}$$

$$SP = \left\{ \frac{5\pi}{16}; \frac{13\pi}{16}; \frac{21\pi}{16}; \frac{29\pi}{16} \right\}$$

$$(9) \quad \cos(2x) = \sin\left(\frac{\pi}{3} - x\right)$$

$$x = \frac{\pi}{6} + 2k\pi \text{ ou } x = -\frac{\pi}{19} + 2k \frac{\pi}{3}$$

$$SP = \left\{ \frac{\pi}{6}; \frac{11\pi}{18}; \frac{23\pi}{18}; \frac{35\pi}{18} \right\}$$

$$(10) \cos 2x = -\cos\left(x - \frac{\pi}{9}\right)$$

$$x = \frac{10\pi}{27} + 2k\frac{\pi}{3} \text{ ou } x = -\frac{10\pi}{9} + 2k\pi$$

$$SP = \left\{ \frac{10\pi}{27}; \frac{28\pi}{27}; \frac{46\pi}{27}; \frac{8\pi}{9} \right\}$$

$$(11) \cos\left(\frac{\pi}{3} - 2x\right) = -\sin\left(x + \frac{\pi}{6}\right)$$

$$x = -\frac{\pi}{9} + 2k\frac{\pi}{3} \text{ ou } x = \pi + 2k\pi$$

$$SP = \left\{ \frac{5\pi}{9}; \frac{11\pi}{9}; \frac{17\pi}{9}; \pi \right\}$$

$$(12) \tan 3x = -\tan\left(2x + \frac{\pi}{4}\right)$$

$$x = -\frac{\pi}{20} + k\frac{\pi}{5}$$

$$SP = \left\{ \frac{3\pi}{20}; \frac{7\pi}{20}; \frac{11\pi}{20}; \frac{3\pi}{4}; \frac{19\pi}{20}; \frac{23\pi}{20}; \frac{27\pi}{20}; \frac{31\pi}{20}; \frac{7\pi}{4}; \frac{39\pi}{20} \right\}$$

$$(13) \sqrt{3} \cdot \sin\left(4x - \frac{\pi}{3}\right) = \frac{3}{2}$$

$$x = \frac{\pi}{6} + k\frac{\pi}{2} \text{ ou } x = \frac{\pi}{4} + k\frac{\pi}{2}$$

$$SP = \left\{ \frac{\pi}{6}; \frac{2\pi}{3}; \frac{7\pi}{6}; \frac{5\pi}{3}; \frac{\pi}{4}; \frac{3\pi}{4}; \frac{5\pi}{4}; \frac{7\pi}{4} \right\}$$

