FONCTIONS TRIGONOMÉTRIQUES

Equations élémentaires

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https://bit.ly/4grikwQ

QVC SD

Résous les équations suivantes en radians, sans calculatrice. Donne également les solutions principales :

(1) $\cos(2x)=\cos(3x-\pi)$

$2x = 3x-\pi+2k\pi$ ou $2x = -(3x-\pi)+2k\pi$

$-x = -\pi+2k\pi$ ou $2x = -3x+\pi+2k\pi$

$x = \pi+2k\pi$ ou $5x = \pi+2k\pi$

$x = \frac{\pi}{5}+2k\frac{\pi}{5}$

$SP = \{\pi ; \frac{\pi}{5} ; \frac{3\pi}{5} ; \frac{7\pi}{5} ; \frac{9\pi}{5}\}$

(2) $2.\sin(2x)-1=0$

$\sin(2x) = \frac{1}{2} = \sin\frac{\pi}{6}$

$2x = \frac{\pi}{6}+2k\pi$ ou $2x = \pi-\frac{\pi}{6}+2k\pi$

$x = \frac{\pi}{12}+k\pi$ ou $2x = \frac{5\pi}{6}+2k\pi$

$x = \frac{\pi}{12}+k\pi$ ou $x = \frac{5\pi}{12}+k\pi$

$SP = \{\frac{\pi}{12} ; \frac{13\pi}{12} ; \frac{5\pi}{12} ; \frac{17\pi}{12}\}$

(3) $4.\tan(3x)+4=0$

$\tan(3x) = -1 = -\tan\frac{\pi}{4} = \tan(-\frac{\pi}{4})$

$3x = -\frac{\pi}{4}+k\pi$

$x = -\frac{\pi}{12}+k\frac{\pi}{3}$

$SP = \{\frac{\pi}{4} ; \frac{7\pi}{12} ; \frac{11\pi}{12} ; \frac{5\pi}{4} ; \frac{19\pi}{12} ; \frac{23\pi}{12}\}$

(4) $\sqrt{2}.\cos\left(3x+\frac{\pi}{2}\right)=1$

$\cos\left(3x+\frac{\pi}{2}\right)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\cos\frac{\pi}{4}$

$3x+\frac{\pi}{2}=\frac{\pi}{4}+2k\pi$ ou $3x+\frac{\pi}{2}=-\frac{\pi}{4}+2k\pi$

$3x=-\frac{\pi}{4}+2k\pi$ ou $3x=-\frac{3\pi}{4}+2k\pi$

$x=-\frac{\pi}{12}+2k\frac{\pi}{3}$ ou $x=-\frac{\pi}{4}+2k\frac{\pi}{3}$

$SP=\left\{\frac{7\pi}{12};\frac{5\pi}{4};\frac{23\pi}{12};\frac{5\pi}{12};\frac{13\pi}{12};\frac{7\pi}{4}\right\}$

(5) $4=5-\tan(2x)$

$\tan(2x)=1=\tan\frac{\pi}{4}$

$2x=\frac{\pi}{4}+k\pi$

$x=\frac{\pi}{8}+k\frac{\pi}{2}$

$SP=\left\{\frac{\pi}{8};\frac{5\pi}{8};\frac{9\pi}{8};\frac{13\pi}{8}\right\}$

(6) $\cos\left(3x+\frac{\pi}{5}\right)=-\frac{1}{2}$ $=-\cos\frac{\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\frac{2\pi}{3}$

$3x+\frac{\pi}{5}=\frac{2\pi}{3}+2k\pi$ ou $3x+\frac{\pi}{5}=-\frac{2\pi}{3}+2k\pi$

$3x=\frac{7\pi}{15}+2k\pi$ ou $3x=\frac{-13\pi}{15}+2k\pi$

$x=\frac{7\pi}{45}+2k\frac{\pi}{3}$ ou $x=\frac{-13\pi}{45}+2k\frac{\pi}{3}$

$SP=\left\{\frac{7\pi}{45};\frac{37\pi}{45};\frac{67\pi}{45};\frac{17\pi}{45};\frac{47\pi}{45};\frac{77\pi}{45}\right\}$

(7) $\sin\left(x-\frac{\pi}{2}\right)=\cos(5x)=\sin\left(\frac{\pi}{2}-5x\right)$

$x-\frac{\pi}{2}=\frac{\pi}{2}-5x+2k\pi$ ou $x-\frac{\pi}{2}=\pi-\left(\frac{\pi}{2}-5x\right)+2k\pi$

$6x=\pi+2k\pi$ ou $-4x=\pi+2k\pi$

$x=\frac{\pi}{6}+k\frac{\pi}{3}$ ou $x=-\frac{\pi}{4}+k\frac{\pi}{2}$

$SP=\left\{\frac{\pi}{6};\frac{\pi}{2};\frac{5\pi}{6};\frac{7\pi}{6};\frac{3\pi}{2};\frac{11\pi}{6};\frac{\pi}{4};\frac{3\pi}{4};\frac{5\pi}{4};\frac{7\pi}{4}\right\}$

(8) $\sin\left(3x-\frac{5\pi}{4}\right)=\sin\left(\frac{\pi}{6}-x\right)$

$3x-\frac{5\pi}{4}=\frac{\pi}{6}-x+2k\pi$ ou $3x-\frac{5\pi}{4}=\pi-\left(\frac{\pi}{6}-x\right)+2k\pi$

$4x=\frac{17\pi}{12}+2k\pi$ ou $2x=\frac{13\pi}{12}+2k\pi$

$x=\frac{17\pi}{48}+k\frac{\pi}{2}$ ou $x=\frac{13\pi}{24}+k\pi$

$SP=\left\{\frac{17\pi}{48};\frac{41\pi}{48};\frac{65\pi}{48};\frac{89\pi}{48};\frac{13\pi}{24};\frac{37\pi}{24}\right\}$

(9) $\sin(2x)+\sin x=0$

$\sin(2x)=-\sin x=\sin(-x)$

$2x=-x+2k\pi$ ou $2x=\pi+x+2k\pi$

$3x=2k\pi$ ou $x=\pi+2k\pi$

$x=\frac{2k\pi}{3}$

$SP=\left\{0;\frac{2\pi}{3};\pi;\frac{4\pi}{3};\frac{5\pi}{3};\pi\right\}$

(10) $3\tan\left(2x+\frac{\pi}{2}\right)=3$

$\tan\left(2x+\frac{\pi}{2}\right)=1=\tan\frac{\pi}{4}$

$2x+\frac{\pi}{2}=\frac{\pi}{4}+k\pi$

$2x=-\frac{\pi}{4}+k\pi$

$x=-\frac{\pi}{8}+k\frac{\pi}{2}$

$SP=\left\{\frac{3\pi}{8};\frac{7\pi}{8};\frac{11\pi}{8};\frac{15\pi}{8}\right\}$

(11) $\sin\left(-2x+\frac{5\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $= \sin\frac{\pi}{3}$

$-2x+\frac{5\pi}{6}=\frac{\pi}{3}+2k\pi$ ou $-2x+\frac{5\pi}{6}=\pi-\frac{\pi}{3}+2k\pi$

$-2x=-\frac{\pi}{2}+2k\pi$ ou $-2x=-\frac{\pi}{6}+2k\pi$

$x=\frac{\pi}{4}+k\pi$ ou $x=\frac{\pi}{12}+k\pi$

$SP=\left\{\frac{\pi}{4};\frac{5\pi}{4};\frac{\pi}{12};\frac{13\pi}{12}\right\}$

(12) $3\tan\left(\frac{\pi}{6}-2x\right)-\sqrt{3}=0$

$x=\frac{k\pi}{2}$

$SP=\left\{0;\frac{\pi}{2};\pi;\frac{3\pi}{2}\right\}$

(13) $\sin\left(3x+\frac{\pi}{6}\right)-\cos\left(x+\frac{4\pi}{3}\right)=0$

$x=-\frac{\pi}{4}+k\frac{\pi}{2}$ ou $x=\frac{5\pi}{6}+k\pi$

$SP=\left\{\frac{\pi}{4};\frac{3\pi}{4};\frac{5\pi}{4};\frac{7\pi}{4};\frac{5\pi}{6};\frac{11\pi}{6}\right\}$