FONCTIONS TRIGONOMÉTRIQUES

Equations non élémentaires

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https://bit.ly/4grikwQ

QVC SD

Résous les équations suivantes en radians, sans calculatrice. Donne également les solutions principales :

(1) $\frac{\sqrt{3}}{2}\cos(2x)+\frac{1}{2}\sin(2x)=\cos\frac{\pi}{7}$

$\cos\frac{\pi}{6}\cdot\cos(2x)+\sin\frac{\pi}{6}\cdot\sin(2x)=\cos\frac{\pi}{7}$

$\cos\left(\frac{\pi}{6}-2x\right)=\cos\frac{\pi}{7}$

$\frac{\pi}{6}-2x=\frac{\pi}{7}+2k\pi$ ou $\frac{\pi}{6}-2x=-\frac{\pi}{7}+2k\pi$

$-2x=\frac{-\pi}{42}$ ou $-2x=\frac{-13\pi}{42}+2k\pi$

$x=\frac{\pi}{84}+k\pi$ ou $x=\frac{13\pi}{84}+k\pi$

$SP=\left\{\frac{\pi}{84}\,;\,\frac{85\pi}{84}\,;\,\frac{13\pi}{84}\,;\,\frac{97\pi}{84}\right\}$

(2) $2\sin^2 x+4\sin x+2=0$

On pose $y=\sin x$ et on a $2y^2+4y+2=0$.

$\Delta=4^2-4.2.2=0$

$y=\frac{-4}{4}=-1$

$\rightarrow \sin x=-1=\sin\frac{3\pi}{2}$

$x=\frac{3\pi}{2}+2k\pi$ ou $x=\pi-\frac{3\pi}{2}+2k\pi$

$x=\frac{3\pi}{2}+2k\pi$ ou $x=-\frac{\pi}{2}+2k\pi$

$SP=\left\{\frac{3\pi}{2}\right\}$

(3) $\sin^2 x-\cos^2 x+\sin x=0$

$1-\sin^2 x$

$\sin^2 x-1+\sin^2 x+\sin x=0$

$2\sin^2 x+\sin x-1=0$

$\Delta=1-4.2.(-1)=9$

$\sin x=\frac{-1\pm 3}{4}$ $\begin{cases}1/2\\-1\end{cases}$

$\rightarrow \sin x=\frac{1}{2}=\sin\frac{\pi}{6}$ ou $\sin x=-1=\sin\frac{3\pi}{2}$

$x=\frac{\pi}{6}+2k\pi$ ou $x=\pi-\frac{\pi}{6}+2k\pi$

$x=\frac{\pi}{6}+2k\pi$ ou $x=\frac{5\pi}{6}+2k\pi$

$x=\frac{3\pi}{2}+2k\pi$ ou $x=-\frac{\pi}{2}+2k\pi$

$SP=\left\{\frac{\pi}{6}\,;\,\frac{5\pi}{6}\,;\,\frac{3\pi}{2}\right\}$

(4) $\cos x+\cos 5x=\cos 3x+\cos 7x$

$2\cdot\cos\left(\frac{x+5x}{2}\right)\cdot\cos\left(\frac{x-5x}{2}\right)=2\cdot\cos\left(\frac{3x+7x}{2}\right)\cdot\cos\left(\frac{3x-7x}{2}\right)$

$2\cdot\cos 3x\cdot\cos(-2x)=2\cos 5x\cdot\cos(-2x)$

$\cos 3x\cdot\cos(2x)=\cos 5x\cdot\cos(2x)$

$\cos 3x\cdot\cos(2x)-\cos 5x\cdot\cos(2x)=0$

$\cos(2x)\cdot(\cos 3x-\cos 5x)=0$

$\cos(2x)=0$ ou $\cos 3x-\cos 5x=0$

$2x=\frac{\pi}{2}+k\pi$; $-2\cdot\sin\left(\frac{3x+5x}{2}\right)\cdot\sin\left(\frac{3x-5x}{2}\right)=0$

$x=\frac{\pi}{4}+k\frac{\pi}{2}$; $-2\sin 4x\cdot\sin(-x)=0$

$4x=k\pi$ ou $-x=k\pi$

$x=k\frac{\pi}{4}$ ou $x=k\pi$

$SP=\left\{\frac{\pi}{4};\frac{3\pi}{4};\frac{\pi}{2};\pi;\frac{5\pi}{4};\frac{3\pi}{2};\frac{7\pi}{4};0\right\}$

(5) $2\cos^2 x+3\cos(x+\pi)=2$

$2\cos^2 x-3\cos x-2=0$

$\Delta=(-3)^2-4\cdot 2\cdot(-2)=25$

$\cos x=\frac{3\pm 5}{4}$ $\begin{cases}2\\ -\frac{1}{2}\end{cases}$

$\rightarrow \cos x=2$ impossible

ou $\cos x=-\frac{1}{2}=-\cos\frac{\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=\cos\frac{4\pi}{3}$

$x=\frac{4\pi}{3}+2k\pi$ ou $x=-\frac{4\pi}{3}+2k\pi$

$SP=\left\{\frac{4\pi}{3};\frac{2\pi}{3}\right\}$

(6) $\sqrt{3}\cos 2x+\sin 2x=2$

$\sin(2x)+\sqrt{3}\cdot\cos(2x)=2$ On pose $\tan\varphi=\sqrt{3}\leftrightarrow\varphi=\frac{\pi}{3}$

$\sin(2x)+\frac{\sin\varphi}{\cos\varphi}\cdot\cos(2x)=2$

$\cos\varphi\cdot\sin(2x)+\sin\varphi\cdot\cos(2x)=2$

$\sin(2x+\varphi)=2$

impossible!

$SP=\emptyset$

(7) $\cos x + \cos 2x + \cos 3x = 0$

$2.\cos\left(\frac{x+3x}{2}\right).\cos\left(\frac{x-3x}{2}\right) + \cos 2x = 0$

$2.\cos(2x).\cos(-x) + \cos(2x) = 0$

$\cos(2x).(2.\cos x + 1) = 0$

$2x = \frac{\pi}{2} + k\pi$

$x = \frac{\pi}{4} + k\frac{\pi}{2}$

$\cos x = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) = \cos\frac{4\pi}{3}$

$x = \frac{4\pi}{3} + 2k\pi$ ou $x = -\frac{4\pi}{3} + 2k\pi$

$SP = \left\{\frac{\pi}{4}; \frac{3\pi}{4}; \frac{5\pi}{4}; \frac{7\pi}{4}; \frac{4\pi}{3}; \frac{2\pi}{3}\right\}$

(8) $\cos 3x + \cos 7x = 1 + \cos 10x$

Carnot : $1 + \cos 2a = 2\cos^2 a$

$2.\cos\left(\frac{3x+7x}{2}\right).\cos\left(\frac{3x-7x}{2}\right) = 2.\cos^2(5x)$

$2.\cos(5x).\underbrace{\cos(-2x)}_{=\cos(2x)} - 2.\cos^2(5x) = 0$

$2\cos(5x).(\cos(2x) - \cos(5x)) = 0$

$5x = \frac{\pi}{2} + k\pi$

$x = \frac{\pi}{10} + k\frac{\pi}{5}$

$\cos(2x) = \cos(5x)$

$2x = 5x + 2k\pi$ ou $2x = -5x + 2k\pi$

$-3x = 2k\pi$ ou $7x = 2k\pi$

$x = \frac{2k\pi}{3}$ ou $x = 2k\frac{\pi}{7}$

$SP = \left\{\frac{\pi}{10}; \frac{3\pi}{10}; \frac{\pi}{2}; \frac{7\pi}{10}; \frac{9\pi}{10}; \frac{11\pi}{10}; \frac{13\pi}{10}; \frac{3\pi}{10}; \frac{17\pi}{10}; \frac{19\pi}{10}; 0; \frac{2\pi}{3}; \frac{4\pi}{3}; \frac{2\pi}{7}; \frac{4\pi}{7}; \frac{6\pi}{7}; \frac{8\pi}{7}; \frac{10\pi}{7}; \frac{12\pi}{7}\right\}$

(9) $\sin^3 x + 2.\cos^3 x = 3.\sin^2 x.\cos x$ *(UCL, Juillet 2018)*

$: \cos^3 x$

$\frac{\sin^3 x}{\cos^3 x} + 2 = 3\,\frac{\sin^2 x}{\cos^2 x}$

$\tan^3 x + 2 = 3\tan^2 x$

$\tan^3 x - 3\tan^2 x + 2 = 0$

On fait $y = \tan x$ et l'équation devient $y^3 - 3y^2 + 2 = 0$

	1	−3	0	2
1	↓	1	−2	−2
	1	−2	−2	0

$\Leftrightarrow (y-1).(y^2 - 2y - 2) = 0$

$y = 1$

$\tan x = 1$

$x = \frac{\pi}{4} + k\pi$

$\Delta = 4 + 8 = 12$

$y = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$

$\tan x = 1 + \sqrt{3}$ ou $\tan x = 1 - \sqrt{3}$

$x = 1{,}22 + k\pi$ ou $x = -0{,}63$

$SP = \left\{\frac{\pi}{4}; \frac{5\pi}{4}; 1{,}22; 4{,}36; 2{,}51; 5{,}65\right\}$

(10) $\sin^3 x.\cos x - \sin x.\cos^3 x = 0$

$\sin x.\cos x.(\sin^2 x - \cos^2 x) = 0$

$\sin x.\cos x.(\sin x - \cos x).(\sin x + \cos x) = 0$

$x = k\pi$; $x = \frac{\pi}{2} + k\pi$

$\sin x = \cos x$

$\sin x = \sin\left(\frac{\pi}{2} - x\right)$

$x = \frac{\pi}{2} - x + 2k\pi$ ou $x = \pi - \left(\frac{\pi}{2} - x\right) + 2k\pi$

$2x = \frac{\pi}{2} + 2k\pi$ ou $x = \frac{\pi}{2} + x + 2k\pi$

$x = \frac{\pi}{4} + k\pi$; impossible

$\sin x = -\cos x$

$\sin x = -\sin\left(\frac{\pi}{2} - x\right)$

$\sin x = \sin\left(-\frac{\pi}{2} + x\right)$

$x = -\frac{\pi}{2} + x + 2k\pi$ ou $x = \pi - \left(-\frac{\pi}{2} + x\right) + 2k\pi$

impossible ; $x = \frac{3\pi}{2} - x + 2k\pi$

$2x = \frac{3\pi}{2} + 2k\pi$

$x = \frac{3\pi}{4} + k\pi$

$SP = \left\{0 ; \pi ; \frac{\pi}{2} ; \frac{3\pi}{2} ; \frac{\pi}{4} ; \frac{5\pi}{4} ; \frac{3\pi}{4} ; \frac{7\pi}{4}\right\}$