

FRACTIONS ALGÈBRIQUES

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<https://bit.ly/3V9lgbU>



1. Réduis les fractions au même dénominateur (sans préciser les conditions d'existence) :

$$(1) \quad \frac{3}{x} + \frac{2}{x^2} = \frac{3x+2}{x^2}$$

$$(2) \quad \frac{x+2}{x} - \frac{x-1}{x+1} = \frac{(x+2)(x+1) - (x-1)x}{x(x+1)} = \frac{x^2+x+2x+2 - x^2+x}{x(x+1)} = \frac{4x+2}{x(x+1)}$$

$$(3) \quad \frac{3x-1}{x+1} + 2 = \frac{3x-1+2(x+1)}{x+1} = \frac{3x-1+2x+2}{x+1} = \frac{5x+1}{x+1}$$

$$(4) \quad \frac{a+b}{a} - \frac{a-b}{b} = \frac{(a+b)b - (a-b)a}{ab} = \frac{ab+b^2-a^2+ab}{ab} = \frac{b^2-a^2+2ab}{ab}$$

$$(5) \quad \frac{4}{a^2b} - \frac{2}{ab^2} = \frac{4b-2a}{a^2b^2}$$

$$(6) \quad \frac{3}{x^2-1} + \frac{2}{(x-1)^2} = \frac{3}{(x-1)(x+1)} + \frac{2}{(x-1)^2} = \frac{3(x-1)+2(x+1)}{(x+1)(x-1)^2} \\ = \frac{3x-3+2x+2}{(x+1)(x-1)^2} = \frac{5x-1}{(x+1)(x-1)^2}$$

$$(7) \quad \frac{x+1}{x^2-4x+4} + \frac{3}{x-2} = \frac{x+1+3(x-2)}{(x-2)^2} = \frac{x+1+3x-6}{(x-2)^2} = \frac{4x-5}{(x-2)^2}$$

$$(8) \frac{\frac{x^2+1}{x^2-x} - \frac{2}{x-1}}{x \cdot (x-1)} = \frac{x^2+1-2x}{x \cdot (x-1)} = \frac{x^2-2x+1}{x \cdot (x-1)} = \frac{(x-1)^2}{x \cdot \cancel{(x-1)}} = \frac{x-1}{x}$$

$$(9) \frac{\frac{a+1}{a^2-a-12} + \frac{2}{a^2+5a+6}}{(a-4)(a+3)(a+2)} = \frac{(a+1)(a+2) + 2(a-4)}{(a-4)(a+3)(a+2)} = \frac{a^2+2a+a+2+2a-8}{(a-4)(a+3)(a+2)} = \frac{a^2+4a-6}{(a-4)(a+3)(a+2)}$$

$$(10) \frac{\frac{1}{a^2-ab} + \frac{1}{ab-b^2}}{a(a-b)b(a-b)} = \frac{b+a}{ab(a-b)}$$

$$(11) \frac{\frac{3}{x^2-1} + \frac{2}{(x-1)^2}}{(x-1)(x+1)} = \frac{3 \cdot (x-1) + 2 \cdot (x+1)}{(x-1)^2 \cdot (x+1)} = \frac{3x-3+2x+2}{(x-1)^2 \cdot (x+1)} = \frac{5x-1}{(x-1)^2(x+1)}$$

2. Simplifie les fractions suivantes (sans préciser les conditions d'existence et de simplification) :

$$(1) \frac{6 \cdot \cancel{x^3}}{\cancel{12} x^2} = \frac{x}{2}$$

$$(2) \frac{4x-8}{2x-4} = \frac{4 \cdot (x-2)}{2 \cdot (x-2)} = 2$$

$$(3) \frac{2x+4}{x^2-4} = \frac{2 \cdot (x+2)}{(x-2)(x+2)} = \frac{2}{x-2}$$

$$(4) \frac{\frac{5}{2} \cdot 10x^2y}{4z} \cdot \frac{8z^2}{9xy^2} = \frac{20xz}{9y}$$

$$(5) \frac{a^2 - b^2}{ab} \cdot \frac{a^2}{a-b} = \frac{(a-b)(a+b)}{b} \cdot \frac{a}{a-b} = \frac{a(a+b)}{b}$$

$$(6) \frac{4a^2 - 9}{2a^2 + 3a} \cdot \frac{a^2}{2a-3} = \frac{(2a-3)(2a+3)}{a(2a+3)} \cdot \frac{a^2}{2a-3} = a$$

$$(7) \left(\frac{1}{x} + \frac{1}{y} \right) \cdot \frac{xy}{x+y} = \frac{y+x}{xy} \cdot \frac{xy}{x+y} = 1$$

$$(8) \left(\frac{x}{x-1} - 1 \right) \cdot \left(\frac{x^2-1}{x} \right) = \frac{x-x+1}{x-1} \cdot \frac{(x-1)(x+1)}{x} = \frac{x+1}{x}$$

$$(9) \frac{x^3 - x}{x^2 + x} = \frac{x(x^2-1)}{x(x+1)} = \frac{x(x-1)(x+1)}{x(x+1)} = x-1$$

$$(10) \frac{x^3 - 3x^2 + 4}{(x-2)^2} = \frac{(x-2)(x^2-x-2)}{(x-2)^2} = \frac{(x-2)(x+1)}{(x-2)} = x+1$$

⑦

	1	-3	0	4
2	↓	2	-2	-4
	1	-1	-2	0

$$x^2 - x - 2 = 0$$

$$\Delta = (-1)^2 - 4 \cdot 1 \cdot (-2) = 9$$

$$x_{1,2} = \frac{1 \pm 3}{2} \begin{matrix} 2 \\ -1 \end{matrix}$$

$$(11) \frac{x^3 + 2x^2 - x - 2}{x^3 - 3x + 2} = \frac{(x-1)(x^2 + 3x + 2)}{(x-1)(x^2 + x - 2)} = \frac{(x+1)(x+2)}{(x-1)(x+2)} = \frac{x+1}{x-1}$$

(N)

$$\begin{array}{c|ccc|c} & 1 & 2 & -1 & -2 \\ 1 & \downarrow & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$\textcircled{D} \begin{array}{c|ccc|c} & 1 & 0 & -3 & 2 \\ 1 & \downarrow & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$$x^2 + 3x + 2 = 0$$

$$\Delta = 9 - 8 = 1$$

$$x = \frac{-3 \pm 1}{2} \begin{cases} -1 \\ -2 \end{cases}$$

$$x^2 + x - 2 = 0$$

$$\Delta = 1 + 8 = 9$$

$$x = \frac{-1 \pm 3}{2} \begin{cases} 1 \\ -2 \end{cases}$$

$$(12) \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{x-1}$$

$$(13) \frac{\frac{2}{x+1}}{1 - \frac{1}{x+1}} = \frac{\frac{2}{x+1}}{\frac{x+1-1}{x+1}} = \frac{2}{x}$$

$$(14) \frac{1 + \frac{2}{x} - \frac{3}{x^2}}{1 - \frac{1}{x^2}} = \frac{\frac{x^2 + 2x - 3}{x^2}}{\frac{x^2 - 1}{x^2}} = \frac{(x-1)(x+3)}{(x-1)(x+1)} = \frac{x+3}{x+1}$$

(N) $x^2 + 2x - 3 = 0$

$$\Delta = 4 + 12 = 16$$

$$x = \frac{-2 \pm 4}{2} \begin{cases} 1 \\ -3 \end{cases}$$

$$(15) \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{y^2 - x^2}{x^2 y^2}}{\frac{y+x}{xy}} = \frac{\frac{(y-x)(y+x)}{xy}}{\frac{y+x}{xy}} = \frac{y-x}{xy}$$

$$(16) \frac{\frac{a^2+b^2}{ab} - 2}{\frac{a}{b} - \frac{b}{a}} = \frac{\frac{a^2+b^2-2ab}{ab}}{\frac{a^2-b^2}{ab}} = \frac{\frac{(a-b)^2}{ab}}{\frac{(a-b)(a+b)}{ab}} = \frac{a-b}{a+b}$$

$$(17) 1 - \frac{1}{1 - \frac{1}{a+1}} = 1 - \frac{1}{\frac{a+1-1}{a+1}} = 1 - \frac{a+1}{a} = \frac{a-(a+1)}{a} = \frac{-1}{a}$$

3. Ecris sous forme d'une puissance de a les expressions suivantes (toutes sensées être définies) ; donne ensuite une réponse sans exposant fractionnaire, ni négatif.

$$(1) \frac{a^{-4}}{a^{-7}} = a^{-4+7} = a^3$$

$$(2) \sqrt[3]{a} \cdot \sqrt{a^3} = a^{\frac{1}{3}} \cdot a^{\frac{3}{2}} = a^{\frac{1}{3} + \frac{3}{2}} = a^{\frac{11}{6}} = \sqrt[6]{a^{11}}$$

$$(3) \quad a^{\frac{3}{2}} \cdot a^{\frac{1}{4}} = a^{-\frac{3}{2} - \frac{1}{4}} = a^{-\frac{7}{4}} = \frac{1}{\sqrt[4]{a^7}}$$

$$(4) \quad \frac{(a^2)^3 \cdot a^{-4}}{a^{-1}} = \frac{a^6 \cdot a^{-4}}{a^{-1}} = a^{6-4+1} = a^3$$

$$(5) \quad (a^2 \cdot \sqrt{a})^3 = (a^2 \cdot a^{1/2})^3 = (a^{2+1/2})^3 = (a^{5/2})^3 = a^{15/2} = \sqrt{a^{15}}$$

$$(6) \quad \frac{\sqrt{a} \cdot a^{-2}}{a^{\frac{3}{2}}} = \frac{a^{1/2} \cdot a^{-2}}{a^{3/2}} = a^{1/2 - 2 - 3/2} = a^{-3} = \frac{1}{a^3}$$

$$(7) \quad \left(\frac{a^{-3} \cdot a^{\frac{1}{2}}}{a^2} \right)^{-1} = \frac{a^2}{a^{-3} \cdot a^{1/2}} = a^{2+3-\frac{1}{2}} = a^{\frac{9}{2}} = \sqrt{a^9}$$

$$(8) \quad a \cdot \frac{\left(a^{\frac{1}{2}} \right)^3}{\sqrt[4]{a}} = a \cdot \frac{a^{-\frac{3}{2}}}{a^{1/4}} = a^{1-\frac{3}{2}-\frac{1}{4}} = a^{-3/4} = \frac{1}{\sqrt[4]{a^3}}$$

$$(9) \quad \sqrt{\frac{a^3 \cdot \sqrt[3]{a}}{a^{-1}}} = \left(\frac{a^3 \cdot a^{1/3}}{a^{-1}} \right)^{1/2} = \left(a^{3+1/3+1} \right)^{1/2} = \left(a^{13/3} \right)^{1/2} \\ = a^{\frac{13}{3} \cdot \frac{1}{2}} = a^{\frac{13}{6}} = \sqrt[6]{a^{13}}$$

$$(10) \left(\frac{\sqrt[3]{a^2} \cdot a^{-\frac{1}{5}}}{\sqrt{a \cdot a^{-3}}} \right)^{-2} = \left(\frac{a^{\frac{2}{3}} \cdot a^{-\frac{1}{5}}}{(a^{1-3})^{\frac{1}{2}}} \right)^{-2} = \left(\frac{a^{\frac{2}{3}} \cdot a^{-\frac{1}{5}}}{a^{-1}} \right)^{-2} = (a^{\frac{2}{3} - \frac{1}{5} + 1})^{-2} \\ = (a^{\frac{22}{15}})^{-2} = a^{-\frac{44}{15}} = \frac{1}{\sqrt[15]{a^{44}}}$$

$$(11) \sqrt{a \cdot \sqrt{a} \cdot \sqrt[3]{a}} = (a \cdot (a \cdot a^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}} = (a \cdot (a^{\frac{3}{2}})^{\frac{1}{2}})^{\frac{1}{2}} = (a \cdot a^{\frac{3}{4}})^{\frac{1}{2}} \\ = (a^{\frac{7}{4}})^{\frac{1}{2}} = a^{\frac{7}{8}} = \sqrt[8]{a^7}$$

$$(12) \sqrt[3]{a^2} \cdot \sqrt[4]{a^3} = (a^2 \cdot a^{\frac{3}{4}})^{\frac{1}{3}} = (a^{\frac{11}{4}})^{\frac{1}{3}} = (a^{\frac{11}{12}}) = \sqrt[12]{a^{11}}$$

$$(13) \left(\frac{a^{-\frac{2}{3}} \cdot \sqrt{a}}{\sqrt[3]{a^{-1}}} \right)^6 = \left(\frac{a^{-\frac{2}{3}} \cdot a^{\frac{1}{2}}}{a^{-\frac{1}{3}}} \right)^6 = (a^{-\frac{2}{3} + \frac{1}{2} - \frac{1}{3}})^6 = (a^{-\frac{1}{2}})^6 = a^{-3} = \frac{1}{a^3}$$

$$(14) \frac{\left(a^{\frac{3}{4}} \right)^{-2} \cdot \sqrt[3]{a^4}}{a^{-\frac{1}{6}}} = \frac{a^{-\frac{3}{2}} \cdot a^{\frac{4}{3}}}{a^{-\frac{1}{6}}} = a^{-\frac{3}{2} + \frac{4}{3} - \frac{1}{6}} = a^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{a}}$$

$$(15) \frac{(a^3 \cdot \sqrt{a})^{-2}}{\sqrt[4]{a^{-5} \cdot a^{\frac{1}{2}}}} = \frac{(a^3 \cdot a^{\frac{1}{2}})^{-2}}{(a^{-5} \cdot a^{\frac{1}{2}})^{\frac{1}{4}}} = \frac{(a^{\frac{7}{2}})^{-2}}{(a^{-\frac{9}{2}})^{\frac{1}{4}}} = \frac{a^{-7}}{a^{-\frac{9}{8}}} = a^{-7 + \frac{9}{8}} \\ = a^{-\frac{47}{8}} = \frac{1}{\sqrt[8]{a^{47}}}$$