Calcul intégral

Intégration par parties

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https://bit.ly/3ZF3ThE.

Calcule :

(1) $\int x.\cos(4x)\,dx$

$f = x \longrightarrow f' = 1$

$g' = \cos(4x) \longrightarrow g = \frac{1}{4}\sin(4x)$

$= x.\frac{1}{4}\sin(4x) - \int \frac{1}{4}.\sin(4x)\,dx$

$= \frac{1}{4}x.\sin(4x) + \frac{1}{16}.\sin(4x) + C$

(2) $\int x.e^{-x}dx$

$f = x \longrightarrow f' = 1$

$g' = e^{-x} \longrightarrow g = -e^{-x}$

$= -x.e^{-x} + \int e^{-x}dx$

$= -x.e^{-x} - e^{-x} + C$

(3) $\int 2x^2.e^x dx$

$f = 2x^2 \longrightarrow f' = 4x$

$g' = e^x \longrightarrow g = e^x$

$= 2x^2.e^x - \int 4x.e^x dx$

$f = 4x \longrightarrow f' = 4$

$g' = e^x \longrightarrow g = e^x$

$= 2x^2.e^x - \left(4x.e^x - \int 4e^x dx\right)$

$= 2x^2.e^x - 4x.e^x + 4e^x + C$

(4) $\int x^3.\ln x\, dx$

$f = \ln x \longrightarrow f' = \frac{1}{x}$

$g' = x^3 \longrightarrow g = \frac{x^4}{4}$

$= \frac{x^4}{4}.\ln x - \int \frac{1}{x}.\frac{x^4}{4} dx$

$= \frac{x^4}{4}.\ln x - \frac{1}{4}\int x^3 dx$

$= \frac{x^4}{4}.\ln x - \frac{1}{4}.\frac{x^4}{4} + C$

$= \frac{x^4}{4}.\ln x - \frac{x^4}{16} + C$

(5) $\int \arcsin x \, dx$

$f = \arcsin x \longrightarrow f' = \frac{1}{\sqrt{1-x^2}}$

$g' = 1 \longrightarrow g = x$

$= x \cdot \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$

$= x \arcsin x - \frac{-1}{2} \int -2x \cdot (1-x^2)^{-1/2} dx$

$= x \cdot \arcsin x + \frac{\cancel{1}}{\cancel{2}} \frac{(1-x^2)^{1/2}}{\cancel{1/2}} + C$

$= x \cdot \arcsin x + \sqrt{1-x^2} + C$

(6) $\int \sin^3 x \, dx$

$f = \sin^2 x \longrightarrow f' = 2 \sin x \cdot \cos x$

$g' = \sin x \longrightarrow g = -\cos x$

$= -\sin^2 x \cdot \cos + 2 \cdot (-1) \int -\sin x \cdot \cos^2 x \, dx$

$= -\sin^2 x \cdot \cos x - 2 \frac{\cos^3 x}{3} + C$

(7) $\int \ln(2x+1)\,dx$

$f = \ln(2x+1) \longrightarrow f' = \dfrac{2}{2x+1}$

$g' = 1 \longrightarrow g = x$

$= x.\ln(2x+1) - \int \dfrac{2x}{2x+1}\,dx$

$u = 2x+1 \Leftrightarrow x = \dfrac{u-1}{2}$

par substitution

$dx = \frac{1}{2}\,du$

$= x.\ln(2x+1) - \int \dfrac{2.\frac{u-1}{2}}{u}.\frac{1}{2}\,du$

$= x.\ln(2x+1) - \frac{1}{2}\int \frac{u-1}{u}\,du$

$= x.\ln(2x+1) - \frac{1}{2}\int \left(1 - \frac{1}{u}\right)du$

$= x.\ln(2x+1) - \frac{1}{2}\left(u - \ln|u|\right) + C$

$= x.\ln(2x+1) - \frac{1}{2}u + \frac{1}{2}\ln|u| + C$

$= x.\ln(2x+1) - \frac{1}{2}(2x+1) + \frac{1}{2}\ln|2x+1| + C$

(8) $\int x^2.\arctan x\,dx$

$f = \arctan x \longrightarrow f' = \dfrac{1}{1+x^2}$

$g' = x^2 \longrightarrow g = \dfrac{x^3}{3}$

$= \dfrac{x^3}{3}\arctan x - \int \dfrac{x^3/3}{1+x^2}\,dx$

$= \dfrac{x^3}{3}.\arctan x - \frac{1}{3}\int \dfrac{x^3}{x^2+1}\,dx$

$$\begin{array}{r|l} x^3 & x^2+1 \\ \hline -(x^3+x) & x \\ -x & \end{array}$$

$= \dfrac{x^3}{3}.\arctan x - \frac{1}{3}\int \left(x - \dfrac{x}{x^2+1}\right)dx$

$= \dfrac{x^3}{3}.\arctan x - \frac{1}{3}\int x\,dx + \frac{1}{3}.\frac{1}{2}\int \dfrac{2x}{x^2+1}\,dx$

$= \dfrac{x^3}{3}.\arctan x - \frac{1}{3}\frac{x^2}{2} + \frac{1}{6}\ln|x^2+1| + C$

$= \dfrac{x^3}{3}.\arctan x - \dfrac{x^2}{6} + \dfrac{1}{6}\ln(x^2+1) + C$

↳ x^2+1 est > 0. On peut donc mettre des parenthèses plutôt que des valeurs absolues